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The STAGS Computer Code

B. O. Almroth and F. A. Brogan

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SUMMARY

The purpose of the present report is to describe the computer code STAGS to potential users and to give information about how the code can be obtained. Primarily, STAGS is intended for analysis of shell structures, although it has been extended through the inclusion of springs and beam elements. STAGS includes options for analysis of linear or nonlinear static stress, stability, vibrations and transient response. A few examples of application of the code are presented for further illustration of its scope.

INTRODUCTION

STAGS is a computer code primarily intended for analysis of structural shells. Its development was initiated in response to the need for analysis of the nonlinear behavior of cylindrical shells with cutouts. Subsequently the code has been improved and its scope extended. NASA-Langley has been the main contributor of funds for its development. Under contract with NASA, STAGS has been converted from being more or less a pure research tool into a code that is suitable for use by the public for practical engineering analysis. Suggestions from NASA-Langley have resulted in considerable enhancement of the code and are to some degree the cause of its increasing popularity.

Other contributors to the development of STAGS are the U.S. Air Force (AFFDL, SAMSO), the U.S. Navy (NSRDC, NADC), and Lockheed MSC through the Independent Research Program.

The STAGS program is documented in a User's Manual consisting of three volumes.

The theory on which the code is based and the organization of the code are described in the first volume. The basic equations are valid

for small strains and moderate rotations. All steps in the analysis that are subject to approximation are summarized in the first section of Volume 1.

The second volume contains the instructions for preparation of input data. Advice is given regarding the modeling of the structure, including the choice of grid. Sometimes, especially for nonlinear static and transient analysis, the success of the effort may depend on the proper choice of computational strategy. Therefore, a special section is included which contains advice on strategy and warnings about certain pitfalls. A section on "output diagnosis" contains recommendations on the action to be taken by the user when certain anomalies occur in the results.

Volume 3 contains a set of example solutions. Input data and representative parts of the output are presented for each of the examples. These cases can be used for the purpose of checking the code, for training of personnel in the use of the code or as illustrative examples whenever the user finds the instructions in Volume 2 unclear.

User reaction consistently seems to indicate that the run time with STAGS is surprisingly low in comparison to that of comparable codes. A STAGS input deck is usually compact and time for its preparation is short. In many situations the user is given the option to define the problem by means of user-written subroutines. This makes it possible to input functional relationships. The user-written routines thus extend the generality of the code, and in addition, they can serve as a labor saving device.

In contrast to many structural codes, STAGS gives rigorous solutions to the algebraic nonlinear equation systems arising from the discrete variational analysis. If convergence is obtained, the solution is independent on the size of the load step. Procedures are included for automatic corrections of load or time steps for optimization of the computational efficiency. For static nonlinear and transient response analyses a restart capability is available.

In the following, some features of the STAGS program are discussed in detail. Discussions of the scope of the code and of the solution procedures used apply to the presently available version of STAGS which is referred to as STAGSC. Extensions now in progress for future versions of the program are briefly discussed. In addition, information is given about required computer equipment and about the availability of the code. Finally, a number of examples of practical applications of STAGS are reviewed for further illustration of the scope of the code.

STRUCTURAL CONFIGURATIONS

STAGS is primarily intended for analysis of thin shell structures. These may contain a number of separate shell branches or segments connected to one another along their boundaries.

Reference Surface Geometry

Any region on which it is possible to define a grid system topologically equivalent to an "I-J" or "row-column" mesh can be defined as a shell branch. The reference surface geometry of a shell branch can be specified for a number of standard geometries by the definition of basic dimensions on regular data cards. The standard geometries are:

- . Rectangular plate
- . Quadrilateral plate
- . Annular plate
- . Cylindrical shell
- . Conical shell
- . Spherical shell
- . Toroidal shell
- . Cylindrical or conical shell with elliptic cross-section
- . Paraboloidal shell
- . Ellipsoidal shell
- . Hyperboloidal shell

For shell branches not included above, the geometric properties of the reference surface may be specified in user-written subroutines. The

user is also provided the option to define global coordinate values at a number of points on the reference surface of the shell branch, in which case a spline fit method will be used for interpolation.

Initial shape imperfections (stress free) may be defined in a user-written subroutine. Cutouts in the shell wall may be defined by the user on source data cards.

For additional versatility the following finite elements have been included in STAGS:

- . Axial rod
- . Torsional spring
- . General beam (nonlinear)
- . Triangular plate (nonlinear)

The triangular plate has been included in order to facilitate the definition of the shell geometry. The triangular element may be used to represent a part of the shell structure which it would be difficult or impossible to model with a row and column scheme.

Shell Wall Properties

The shell wall properties are defined by a 6 by 6 local stiffness matrix relating stress and moment resultants to reference surface strains and changes of curvature. For a number of standard types of shell walls the user can read dimensions and material properties on data cards and the shell wall stiffness matrix is internally computed. The standard types of shell walls are:

- . Layered shells; shells may be anisotropic with respect to the shell surface coordinates. However, the user defines the properties in the direction of the principle axes for the material and gives the angle between these directions and the directions of the surface coordinates.

- Fiber-wound layered shells (stiffness computed from properties of constituent parts).
- Corrugated wall or corrugation-stiffened wall.
- Shells with "smeared" stiffeners (see below).

For shell walls not included among the standard types, the local stiffness matrix is provided by means of user-written subroutines. A user-written subroutine is also required when dimensions or material properties vary over the shell surface. In this case, the resident routines for standard wall types can be utilized for computation of the stiffness matrix after dimensions and properties have been defined as functions of the shell coordinates.

Inelastic analysis is limited to isotropic materials and is based either on

- The White-Besseling theory (mechanical sublayer)
- Flow theory with isotropic hardening.

While the elastic properties are allowed to vary with temperature, no provisions have been made to include temperature dependence in plastic deformation.

Stiffeners are defined as beams attached to the shell along some line on its surface. The line of attachment need not coincide with any of the gridlines on the shell surface.

Source data cards can be used for stiffeners with constant cross-section but subroutines must be introduced if dimensions or material properties vary along the stiffener. Stiffeners may be considered as discrete elements or their contributions to the shell wall stiffness properties may be smeared out over the shell surface so that the stiffener-shell wall combination is treated as an equivalent anisotropic shell. Thermal and inelastic effects in the stiffeners are included.

If a structural element is defined as a stiffener, it is assumed that its cross-section does not deform or warp during loading. For more accurate treatment of such elements, they must be considered as separate shell branches.

Displacement Constraints

Displacement constraints may be defined in three different ways.

- Source data cards may be used for definition of boundary conditions if the same conditions prevail along an entire edge of a shell branch. The user is allowed to leave free or suppress any of the three displacement components and the edge rotation. A number of standard boundary conditions, including symmetry and displacement compatibility between branches, are defined by one single value of the control parameter.
- Source data cards can also be used for pointwise displacement constraints, along branch boundaries or internally.
- Constraints that involve more than one displacement (or rotation) component must be defined by user-written subroutine. Any homogeneous linear displacement constraint can be defined this way.

In bifurcation buckling or vibration analysis, it is possible to define constraints for the incremental displacements that are different from the constraints on the basic stress state.

Loads

Mechanical loading may be introduced by definition of either forces or displacements. Point forces, line loads, and uniform surface tractions (including pressure) may be defined by use of source data cards. A completely general system of loads can be defined in this way. User-written subroutines are used then, not for generality but rather for compactness of the input deck. Much labor frequently will be saved by the use of functional relations in the definition of the loading.

Point forces, line loads and surface tractions are generally assumed to maintain their original direction during loading. The only exception to this is that a normal pressure can be declared to be a "live load". In that case, the pressure remains normal to the deformed surface. Temperature loads are always introduced through user-written subroutines.

SOLUTION PROCEDURES

Type of Analysis

In the general case the structural behavior is governed by the equations of motion. A nonlinear transient response analysis then yields answers to all questions about structural behavior, including those of static or dynamic stability. However, it is frequently possible to obtain satisfactory results by use of less expensive means. Consequently, structural analysis usually consists of the application of one or more of a number of special procedures that are based on some simplification of the basic equations. Traditionally, these procedures (stability, vibrations, etc.) have been considered as separate disciplines having little to do with one another. The high speed computer has opened more options to the analyst and in order to make the best possible use of the available computer programs he needs a good understanding of the theory and of the relations between the different types of analyses. These relations are illustrated in a block diagram in Figure 1. The structural behavior is governed by the equation

$$M\ddot{u} + D(\dot{u}) + B(u) + K(u) = F \quad (1)$$

where u is a vector of displacement components, M is the mass matrix, F is a vector of external forces, and K the generally nonlinear stiffness operator. The operators B and D include forces that are functions of structural deformation and deformation velocity, respectively. If such forces are present, as in the case of aerodynamic forces on an airplane wing, the basic equations can generally not be derived from a potential. The problem is not conservative and the resulting equation system is not symmetric.

If loads vary slowly, all time derivatives of the displacement components may be disregarded and a static analysis is sufficient. If that is the case and if the system is conservative, the next question is whether nonlinear terms should be retained. Partly due to economical constraints, the linear static stress analysis has been by far the most commonly used mode of analysis. If geometrical nonlinearities must be accounted for, it is possible to obtain limited but sometimes sufficient information about their effect without actually solving the nonlinear equations. A perturbation technique then is used, in which

$$u = \lambda \underline{u}_0 + \underline{u}_1 \quad (2)$$

is substituted into the nonlinear equations. Here $\underline{u}_0$ represents the solution of the basic equations after omission of nonlinear terms and λ is a load parameter to be determined. The perturbation $\underline{u}_1$ is so small that higher order terms in $\underline{u}_1$ can be discarded. The so-called bifurcation buckling load is represented by the value λ that allows nontrivial solutions to the homogeneous equation system in $\underline{u}_1$. If the precritical behavior is truly linear, the value of λ so computed represents a bifurcation in the load-displacement diagram. The bifurcation buckling analysis can often be used as an approximation also for cases with nonlinear precritical behavior. For certain types of structures, such as shells of revolution, bifurcation from a nonlinear prebuckling path may represent a rigorous solution.

STAGS has not been adopted for solution of general nonconservative problems but otherwise it includes all the analysis procedures discussed above. Nonconservative problems can be handled if they are purely dissipative, i.e., problems with plastic deformation and structural damping. This means that the forms of analyses represented in the boxes of solid lines in Figure 1 are included in STAGS.

Discretization

A number of procedures are available for reduction of a continuum problem into a problem with a finite number of degrees of freedom. With the advent of the high speed computer, methods based on kinematic discretization of the continuum have become predominant. That is, the degrees

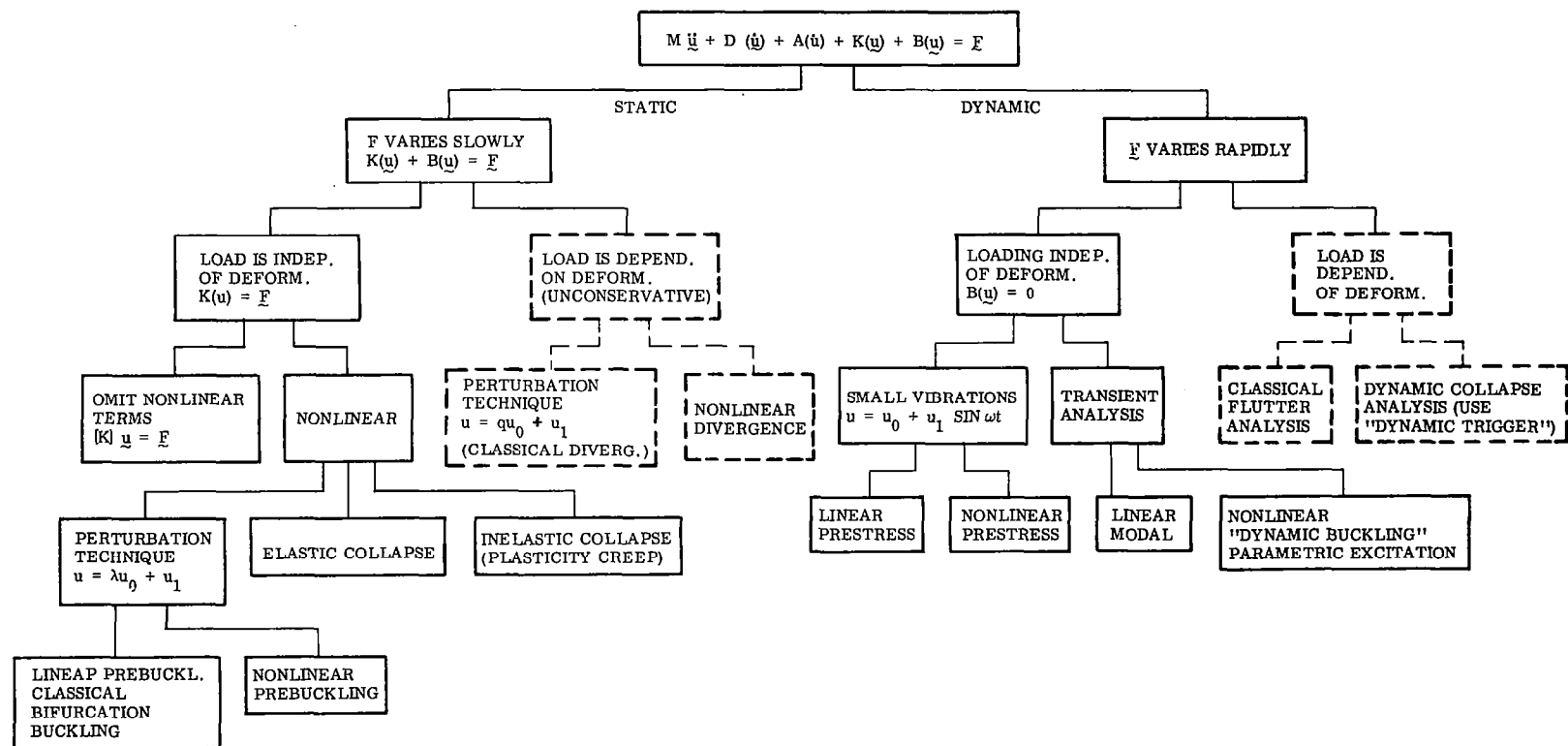


Fig. 1 Summary of Analytical Procedures

of freedom of the system are the displacement components at a finite number of node points. STAGS is based on such a discretization as formulated on a variational basis. By use of local power series approximations for the displacements the density of strain energy is determined at a number of integration points. The total potential energy is then obtained through numerical integration over the structure and subtraction of the work done by external forces. Rotations have been introduced as freedoms at all node points. Consequently, it has been possible to make standard finite element procedures and finite difference energy methods available as options.

Three options are available for generation of node points:

- Node points are located at intersections between lines corresponding to constant values of the basic surface coordinates defining the directions of inplane displacement components.
- If the preceding option is not practical, an independent system of surface coordinates may be used for automatic grid generation.
- As a final alternative, node points may be defined without reference to a set of surface coordinates, point by point through source data cards or by a user-written subroutine.

If surface coordinates are used, it is sufficient to define the position of node points along two boundary lines. Variable spacing is allowed and certain grid irregularities can be specified.

Linear Equation Systems

All modes of analysis except transient response analysis with explicit integration involve the solution of large systems of simultaneous linear equations. Two- or three-dimensional problems result in systems with a large bandwidth in the coefficient matrix. However, this matrix is sparse, i.e., most of its elements are zero. The efficiency of a solution procedure depends largely on the ability to anticipate and thus avoid multiplications in which one factor equals zero. STAGS is based on the "skyline method", that is, full advantage is taken of the

zeros following the last nonzero element in each equation. Other sparse matrix methods and automatic renumbering could possibly improve computational efficiency but are presently not available in STAGS.

Eigenvalue Analysis

Bifurcation buckling and small vibration analyses lead to linear eigenvalue problems. These are solved in STAGSC through the generation of invariant subspaces by simultaneous inverse power iteration (Ref. 1). In vibration analyses a number of eigenvalues are usually required. In that case the subspace method is more efficient than any other method presently in use. For buckling analyses the user sometimes is interested only in the lowest eigenvalue. The subspace method may then be slightly slower than a straight inverse power iteration if this eigenvalue is well separated from the others. However, the subspace method always gives some information about other buckling modes. This information, of course, is valuable if the analysis indicates that some reinforcement of the structure is necessary. The higher eigenvalues may not be noticeably affected by structural modifications introduced exclusively for prevention of buckling in the mode corresponding to the lowest eigenvalue.

Nonlinear Equation Systems

A large number of algorithms are available for solution of nonlinear algebraic systems. In Reference 2 these are classified in four groups:

- . Newton-like methods
- . Method of successive substitution
- . Initial-value methods
- . Direct minimization methods

It is shown in Reference 3 that all the methods frequently used in structural analysis may be considered as special applications of the two Newton-like methods:

- . Regular Newton method
- . Modified Newton method

Based on the present state of the art, it appears sufficient to include these two methods in the code. They are presently included in STAGS and advice is given in the User's Manual regarding the choice between these methods.

Initial Value Problem

In a transient response analysis one can choose between modal superposition and direct integration of the equations of motion, either by explicit or implicit methods. An important difference between the explicit and implicit methods is that the former are mathematically stable only if the time step is sufficiently small, while the latter can maintain numerical stability at much larger time steps. An approximate value for the critical time step in the explicit method is readily computed and if this is not exceeded, the solution will generally be quite accurate for all deformation modes involved in the motion. One problem with the explicit schemes, such as the frequently used central difference scheme, is that it requires mass matrix entries corresponding to the rotational freedoms. Implicit methods are more flexible as they allow the sacrifice of some accuracy in the interest of computer economy.

High order integration methods can sometimes be used to further reduce the run time. However, such an analysis is more difficult because of potential instability associated with spurious roots. Discretized structures generally have a very wide range of vibration frequencies. For analysis of such systems, it appears that the stiffly stable methods developed by Gear (Ref. 4) and later modified by Jensen (Ref. 5) and Park (Ref. 6) are most suitable. The following time integration methods are included in STAGS.

- . The explicit central difference scheme
- . Trapezoidal rule
- . Gear's 2nd and 3rd order methods
- . Park's method

Advice regarding the choice between these methods is included in the User's Manual.

EXTENSIONS IN PROGRESS

Two major extensions are now in progress:

- . The finite element library is being extended by inclusion of a brick element for three dimensional analysis and a "contact-element".
- . Classical Rayleigh-Ritz analysis is introduced as one solution option for certain problems. This is done by a transformation in which the nodal displacements are eliminated as unknowns in favor of the amplitudes of a set of global displacement shape functions. The shape functions can be defined as vibration modes, buckling modes, or be user-written displacement functions. Control on the accuracy is included as well as automatic adjustment of the set of global shape functions in case the solution deviates from the solution of the discrete system.

With respect to computational efficiency, the code appears to be well up to date. Some efficiency, particularly with respect to storage requirements, can be gained by a decomposition of the code into smaller modules. The Rayleigh-Ritz procedure discussed above will probably serve to reduce the computer time dramatically in many cases at the expense of a moderate sacrifice of accuracy.

A research program is in progress at Lockheed Missiles and Space Company, Inc. which includes evaluation of discretization methods and procedures for solution of the algebraic equations. STAGS will be modified in accordance with the results of such investigations to the extent that funding is available.

Beside generality, reliability and computational efficiency, the most important feature of a computer code is ease of use. In this area the following improvements are under development.

- . A provision is made to allow use of source data cards for definition of displacement compatibility between branches along internal gridlines (rather than boundary lines only).
- . A study of the possibilities of automating decisions on strategy in nonlinear and transient analysis.

COMPUTER EQUIPMENT REQUIREMENTS

STAGSC is presently operational on the CDC and on the UNIVAC Exec 8 systems (in single or double precision). A core size of 64 K (decimal) is sufficient for most problems. However, for very large cases, a larger core may be required and improved economy can generally be achieved by use of a larger part of the core. Figure 2 shows how the computer time on the CDC 6600 can be estimated for linear analysis and for bifurcation buckling analysis. The curves are valid for the finite difference procedure. If a higher order (finite element method) is used, the run time for the same grid size, will increase. However, since a much coarser grid can be used, the computational efficiency may still be better for these elements. For a nonlinear analysis it is more difficult to estimate the run time. If the nonlinearity is moderate, it may be possible to obtain convergence directly at the load level of interest (design load). In this case the computer time would be only slightly more (about 20 percent) than for a linear analysis. With considerable nonlinearity one must take several load steps to reach the design load and often factor the matrix (decomposition in upper and lower triangular matrices) more than once. In such cases, the computer time obviously will be several times larger than it is for a linear analysis. If a collapse load (limit point) is to be determined, the run time depends on how severely the shell is deformed before collapse, or how much redistribution of stress takes place. Ordinarily a collapse analysis requires between 10 and 100 times the computer time for linear analysis.

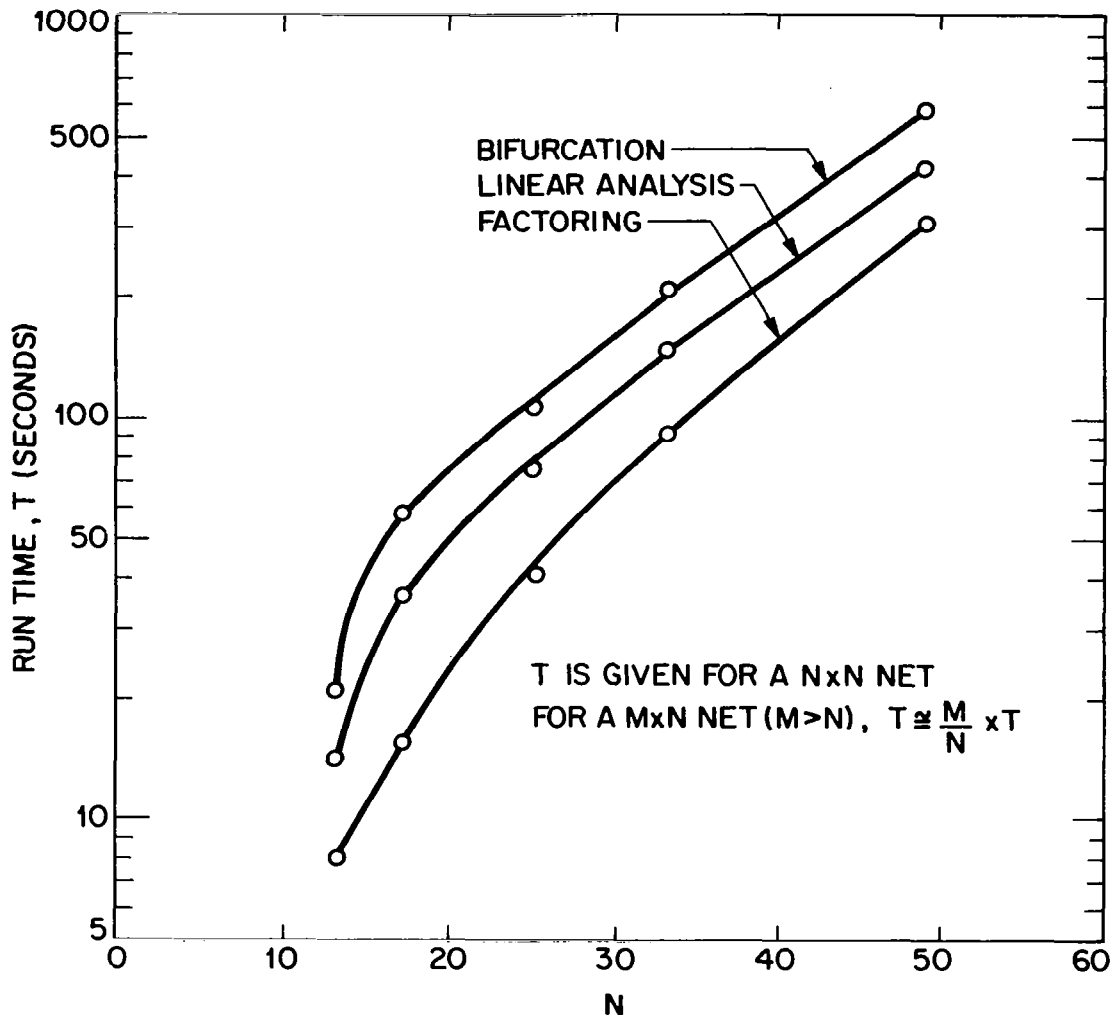


Figure 2 CDC 6600 Computer Run Time For
Finite Difference Nets (N x N)

The basic problem dimensions (number of nodal points in each direction) are restricted primarily by the available mass storage. If the number of nodal points in the two directions equals n and m , where $n \leq m$, an approximate value of the required number of words of

mass storage M is given by

$$M = (k_1 n + k_2)nm$$

where

$$k_1 = \begin{cases} 12 & \text{for linear or nonlinear stress analysis} \\ 24 & \text{for bifurcation buckling analysis} \end{cases}$$

and

$$k_2 = 300 \text{ to } 450 \text{ depending on the complexity of the problem}$$

For a simple geometry (plate, cylinder, cone) and with uniform grid spacing, the minimum value of k_2 can be used. For branched shells and for cases requiring use of finite elements, the constant k_1 is somewhat larger. For inelastic analysis a quantity $6mn(pq + 2)$ must be added in the above equation for required storage where p is the number of integration points through the thickness and q is the number of material components.

The computer time (CDC 6600) and required auxiliary storage space are shown here for two examples.

- A. Linear stress analysis of cylindrical shell with a 30 by 180 grid (16,200 degrees of freedom)

$$k_1 = 12, k_2 = 300$$

$$M = 2,564,000 \text{ words}$$

$$\text{Computer time} = 12 \text{ min. (from Fig. 2)}$$

For inelastic analysis with five integration points through the thickness and with six material components (with the White-Besseling method)

$$M = 3.6 \times 10^6 \text{ words}$$

B. Bifurcation buckling of shell with a nonorthogonal 30 by 180 grid

$$k_1 = 24, k_2 = 450$$

$$M = 6.3 \times 10^6 \text{ words}$$

Computer time = 16 min.

AVAILABILITY OF STAGSC

The STAGSC program can be obtained through

- The Cosmic Library

For information contact:

Computer Software Management and Information
Center (COSMIC)
112 Barrow Hall
University of Georgia
Athens, Georgia 30602

- Lockheed Missiles and Space Company, Inc.

For information contact:

Structures Laboratory
Department 52-33, Building 205
Lockheed Research Laboratories
3251 Hanover Street
Palo Alto, California 94304
Telephone (415) 493-4411 (ext. 45195)

In both cases a nominal fee is charged to cover service expenses.

EXAMPLE CASES

In the following are presented results from some of the applications of STAGS. A number of parametric studies have been undertaken which were designed to yield a better understanding of the behavior of some structural components. These studies include:

- Axially loaded cylindrical shells with cutouts (Refs. 7, 8)
- Long cylinders in bending (Ref. 9)
- Postbuckling behavior of shear panels (Ref. 10)
- Shells with elliptic cross-section (Refs. 11, 12)
- Vibration of plates with nonuniform heating.

Some details from the studies of cylindrical shells with cutouts, of long cylinders under bending, and of the vibration characteristics of heated plates, are presented below.

The STAGS code has also been applied in the analysis of some major components of structural hardware. Many of those studies have been performed by Lockheed Missiles and Space Company, Inc., personnel under subcontract. These include:

- The Skylab shell (Ref. 13)
- A part of the B-1 fuselage (Ref. 14)
- The Centaur stub adaptor (Ref. 15)
- The Viking aero-shell
- Space Shuttle cargo doors

Some details from the Skylab shell and the Space Shuttle cargo door analyses are presented below.

Cylindrical Shells With Cutouts

In the STAGS studies of the collapse of shells with cutouts (Refs. 7, 8), the critical axial load is determined by use of a nonlinear analysis. Figure 3 shows how the normal displacement at the cutout edge varies with the axial load for a cylinder with two diametrically opposite rectangular cutouts. The critical load according to bifurcation buckling theory is indicated in the figure. The initial trend of the lateral displacement at the cutout edge appears to be toward the horizontal asymptote defined by the bifurcation buckling load. However, as the

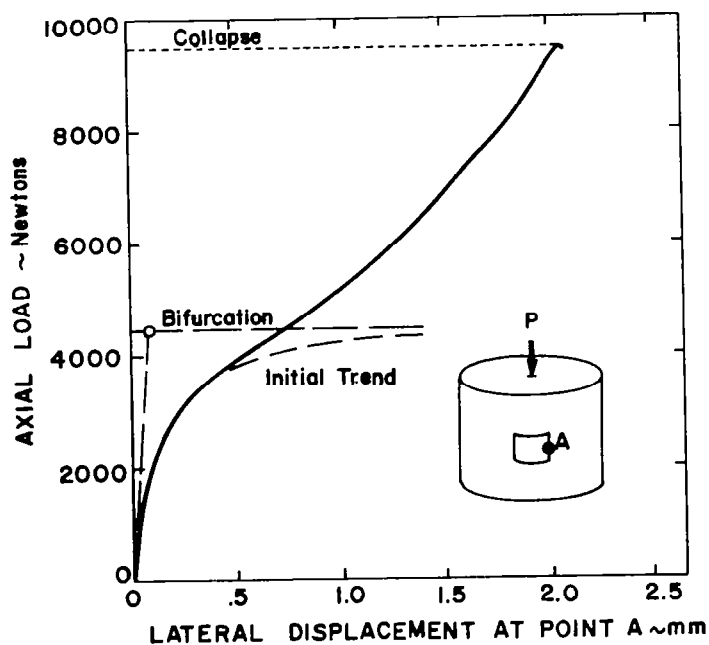


Figure 3 Deformation of Cylinder with Cutouts

stresses are redistributed away from the cutout edge, the slope of the load-displacement curve increases again and collapse does not occur until a load level is reached which is more than twice the load indicated by bifurcation theory.

Under contract with NASA, Johnson Space Center, Houston, analytical results obtained by use of STAGS were compared to experimental results (Ref. 7). Two diametrically opposite cutouts were made in each of the cylinders. The width of the cutout and the shell thickness were varied within the test series. Tests were made with cutouts covering either 30° or 45° .

One of the test specimens with 45° cutouts and with .355 mm thickness was equipped with a large number of strain gages. Consequently, it was possible to make comparisons between measured and computed stresses. Such comparisons are shown in Figure 4. The solid lines represent computed stresses and the points are the stress values determined by the strain gages. The deviations between measured and computed stresses is nowhere more than can be explained by variations in thickness on the test specimens. Measured and computed collapse load for the cylinders with unreinforced cutouts are shown in Table I.

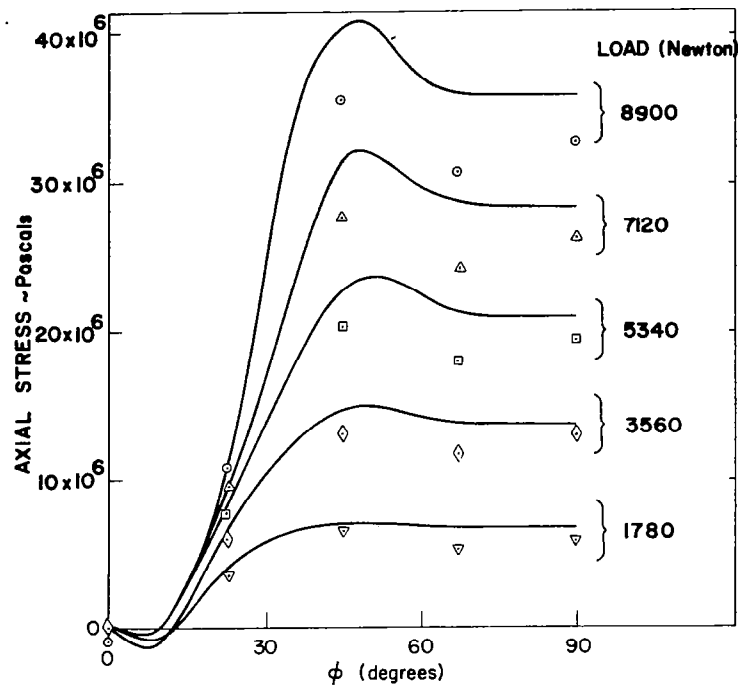


Fig. 4 Measured and Computed Stresses
in Cylinder with Cutouts

TABLE I

Measured and Computed Collapse Loads (Newtons)

Cutout Width	Critical Loads	
	Computed	Measured
30°	13000	12300
45°	10000	10000
45°	10000	8960

The influence of reinforcements around the cutout edges is also studied in Ref. 7. As the reduction of the critical load due to the cutout is diminished with such reinforcement, the effect of random geometrical imperfections is more severe than the effect of the cutout and the agreement between test and theory is not as good as for the cylinders with unreinforced cutouts.

STAGS has also been used for collapse analysis of cylinders with circular cutouts (Ref. 8). Results obtained with STAGS are shown in Figure 5. Experimental and theoretical results are in very good agreement. Here $\bar{P}_{cr}$ is the critical axial load divided by the corresponding load for a cylinder without a cutout.

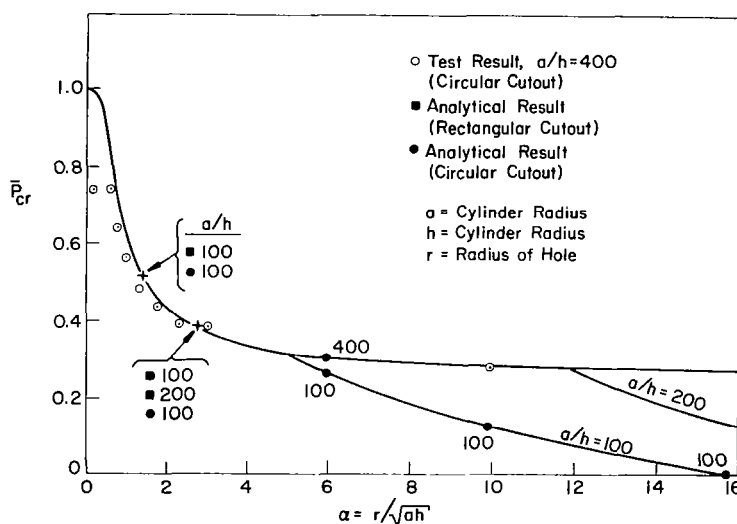


Fig. 5 Critical Loads for Cylinders with Cutouts

Bending of Long Cylindrical Shells

In a curved tube subjected to bending, the cross-section tends to deform because the axial stress resultants acting on a shell element have components directed toward the center of the tube. In an initially straight tube the same effect will occur as a result of the curvature caused by the applied load. With increasing bending moment, the shell flattens, resulting in a reduction of its bending stiffness. A graph showing the bending moment as a function of the applied curvature will display a maximum which corresponds to shell collapse.

This problem was first considered in 1926 by Brazier (Ref. 16). The analysis was applied to infinitely long shells only, and it was assumed that the displacements were small in comparison to the shell radius. Brazier found that the maximum in the load displacement curve is reached when the lateral displacement is

$$w = \frac{2}{9} a .$$

If the bending stress at collapse is computed by use of properties of the undistorted cross-section, and with $\nu = 0.3$, then

$$\sigma_{CR} = 0.33 E(h/a)$$

where h is the wall thickness of the tube.

The problem of stability of circular cylindrical shells under bending was solved as a bifurcation buckling problem by Seide and Weingarten in 1961 (Ref. 17). Assuming that the prebuckling behavior can be defined with sufficient accuracy by a linear membrane solution, they found that the critical buckling stress was only 1.5 percent higher than the critical stress for uniform compression for a shell with $a/h = 100$. For thinner shells the difference is even smaller. Thus, for all practical purposes, the critical stress would be accurately determined by

$$\sigma_{CR} = 0.605 E(h/a)$$

This value is well above the critical stress found by Brazier for infinitely long cylinders. However, boundary conditions usually restrict deformations so that at the shell edges the cross-section remains circular. Therefore, collapse through flattening will occur at a higher load level than predicted unless the shell is very long. For sufficiently short shells the prebuckling behavior is well approximated by the linear membrane solution and, aside from the effects of initial imperfections, the solution is satisfactory. For longer shells there is a coupling between the flattening of the cross-section and the formation of a short-wave buckle pattern as the flattening of the cross-section reduces the load at which the wave pattern appears. By use of STAGS, results can be obtained for shells of intermediate length and some of Brazier's approximations need not be made. Hence, the results are more accurate for very long shells also. The results from the study are summarized in Figure 6. The bending moment and the internal pressure are normalized with respect to their critical values in a bifurcation buckling analysis with a linear stress state.

Vibrations of Heated Plate

For reduction of drag many platelike lifting surfaces are designed with very thin edges. At high velocity aerodynamic heating causes a thermal stress field with spanwise compression at the thin edges. This may have a profound effect on the stiffness of the wing. The STAGS program was used in a study of the effect on the aeroelastic properties of such wings (Ref. 18). Previous efforts in this area were published before the high speed computer made detailed analysis of more complex configurations possible. The analytical solution in Reference 19, for example, uses a temperature distribution typical of a wing with a diamond shaped profile but in the structural analysis it is handled as a plate with constant thickness. Some results of the STAGS analysis are presented here.

The geometry of the wing is shown in Figure 7. The chordwise variation in temperature is given by $T = \bar{T}(2x/c - 1)^4$, where c is the

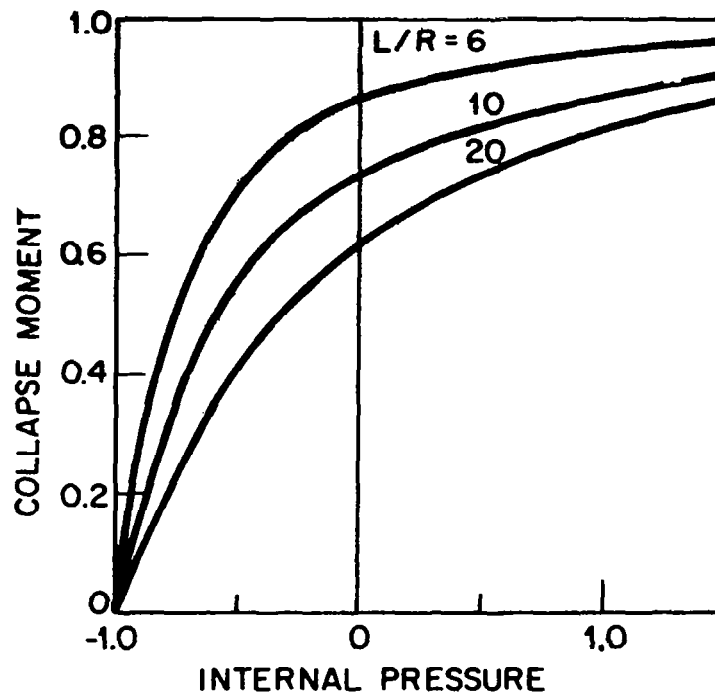


Fig. 6 Stability of Long Cylindrical Shells under Bending

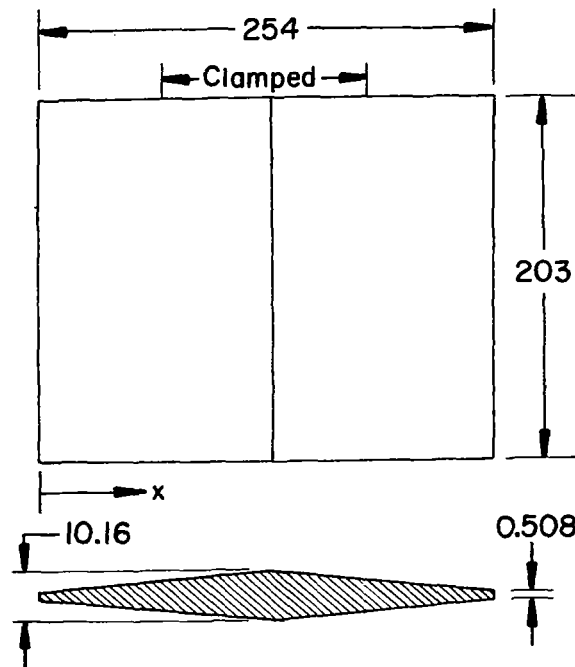


Fig. 7 Geometry of Wing with Diamond-Shaped Profile (dimensions in mm)

length of the chord. As the temperature is increased, a level will be reached at which the axial compressive stresses at the thin edges will cause thermal buckling. A bifurcation buckling analysis shows that this will occur at $\bar{T} = 366^{\circ}\text{C}$. If the temperature is further increased, the buckles are gradually deepening as indicated by the curve marked $P = 0$ in Figure 8. Load displacement curves are shown also for plates with "shape imperfections". These were introduced by the application of a couple of small forces with opposite directions at the leading and trailing edges of the wing. Load displacement curves are shown in Figure 8 for the cases in which these forces are 0.445 and 2.225 N, respectively. The stability matrices corresponding to symmetric (with respect to mid-chord) and antisymmetric modes each have two zero roots very close together at $\bar{T} = 366^{\circ}$. Therefore, we expect two vibration frequencies corresponding to symmetric and two corresponding to antisymmetric modes to vanish when the critical temperature is reached. As the postbuckling configurations correspond to stable equilibrium, these frequencies should again increase when the temperature is raised above its critical value. The five lowest frequencies for symmetric modes are shown in Figure 9. Symmetric vibration modes at a number of selected values of $\bar{T}/T_{\text{CRIT}}$ are shown in Figure 10. For comparison the two buckling modes corresponding to the lowest eigenvalues are also shown.

If a small geometric imperfection is present or if small lateral forces are applied, there will not be a bifurcation in the load displacement curve and consequently no temperature value at which the vibration frequencies will vanish. Frequencies for the two "critical symmetric modes" are shown in Figure 11.

The critical modes are local in character. In addition, a very small imperfection is sufficient to make the lowest frequencies relatively insensitive to the occurrence of thermal buckling. Therefore, thermal buckling is not as severe from an aeroelastic point of view as is indicated by results from previous investigations.

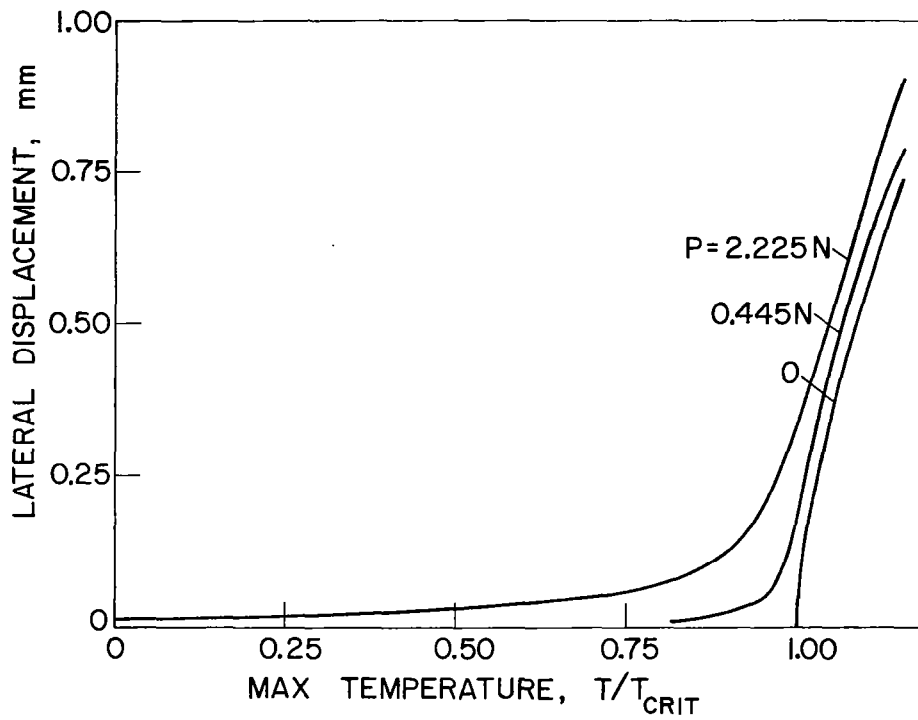


Fig. 8 Lateral Displacement Versus Temperature

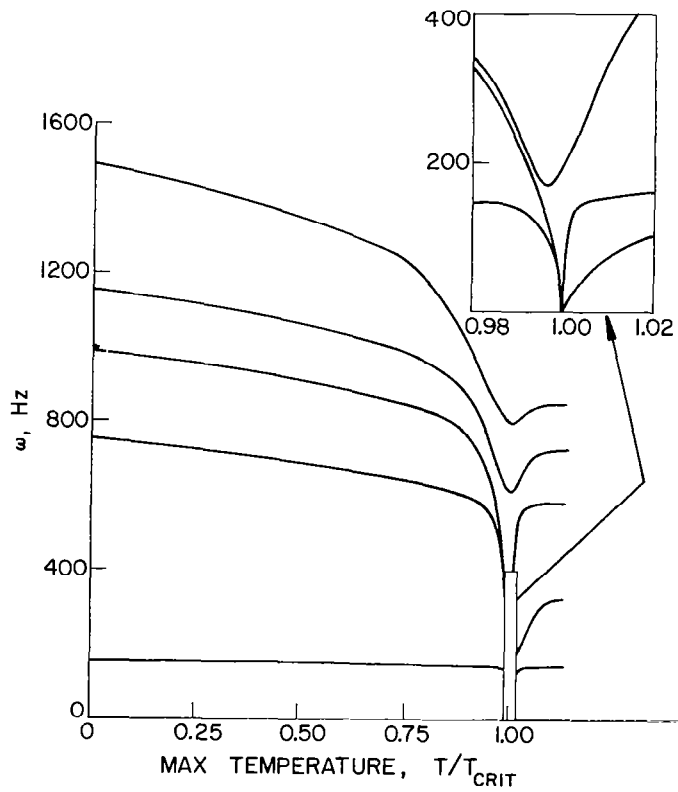
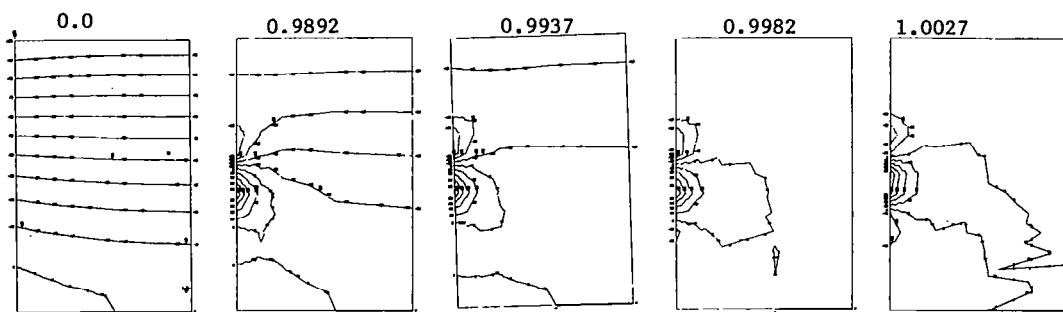
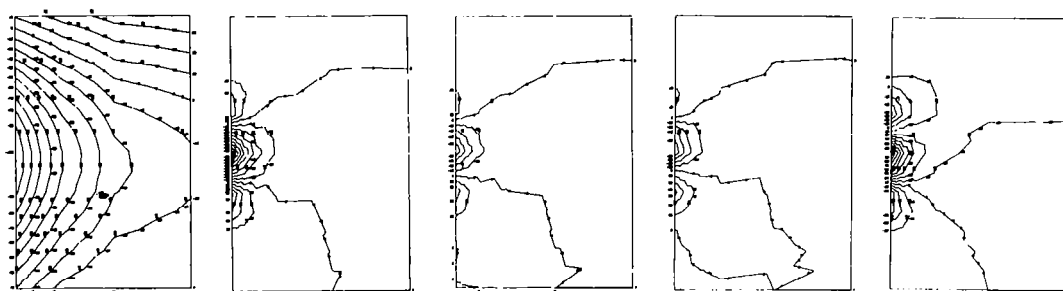


Fig. 9 Frequencies of Symmetric Modes Versus Temperature

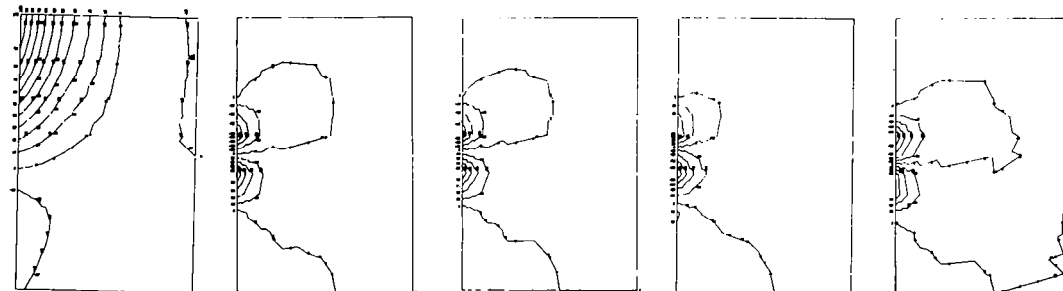
$$T/T_{crit} =$$



1. Mode



2. Mode



3. Mode

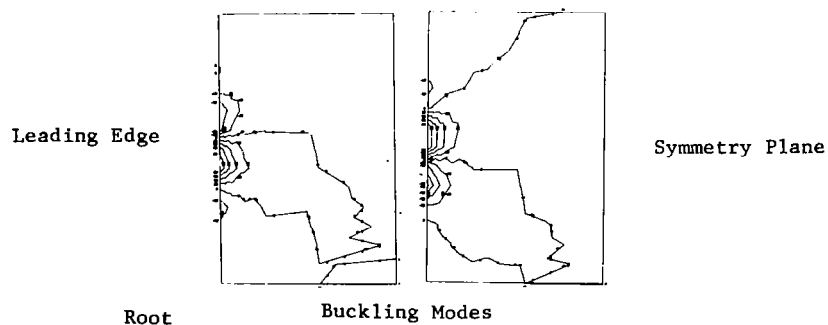


Fig. 10 Symmetric Vibration and Buckling Modes

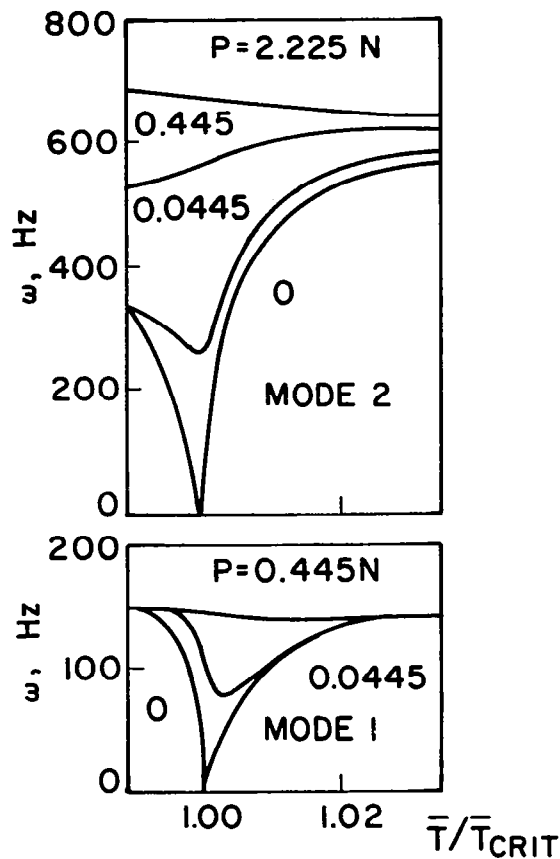


Fig. 11 Vibration Frequencies for Imperfect Plates

Stability Analysis of the Skylab Structure

Reference 13 describes an analysis of the Skylab structure which consists of a series of ring-stiffened cylindrical shells. Two internal payloads are each supported at four points along the circumference. These concentrated loads are introduced into the shell structure through four longerons. Hence, neither structure nor loading is axially symmetric and a two-dimensional analysis must be used. As general instability may be critical, rather than a localized buckling pattern, it is desirable that the entire structure is treated in a buckling analysis and that the question of applicability of the bifurcation buckling analysis be carefully considered. The attachments of payloads and the structural arrangements are shown in Figure 12.

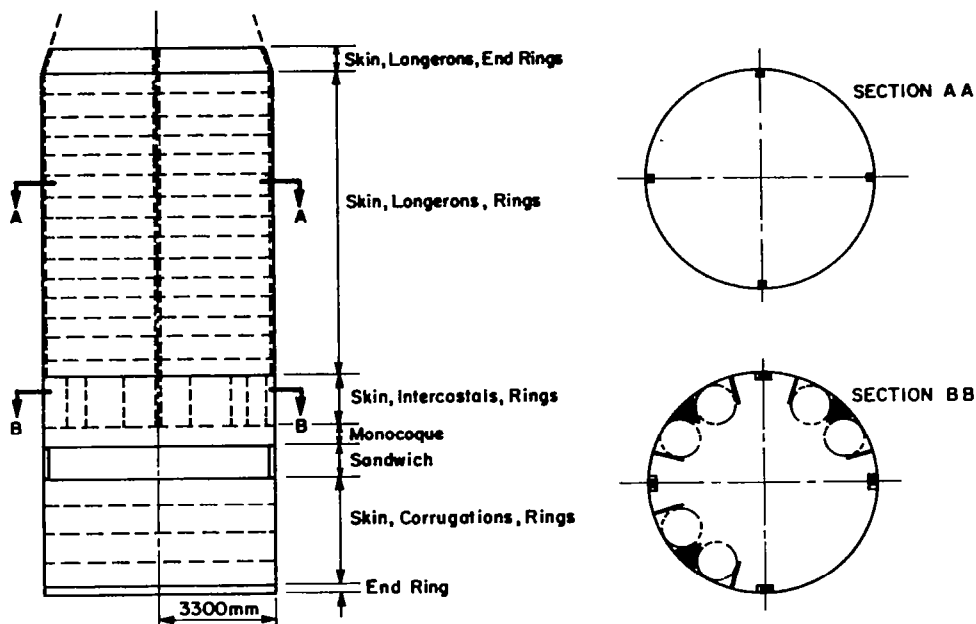


Fig. 12 The Skylab Structure

Many load cases were considered and extensive convergence studies were carried out to make certain that a sufficiently fine grid was used. After the convergence study had been completed, the most critical load cases were analyzed by use of a model covering 180° of the shell structures (mesh size 39 by 168). This analysis involved more than 20,000 degrees of freedom. Some data from the analysis are shown in Table II. The estimate of the run time (see Figure 2) indicated that the case might run as long as one hour. However, Table II shows a run time of only 40 minutes indicating that the estimate may be conservative for very large cases.

There may be some questions about the applicability of the bifurcation buckling theory in a case like this. Therefore, a nonlinear analysis was performed. For computer economy, this was based on a somewhat smaller model (mesh size 33 by 108). A geometrical imperfection was included in this run which was approximately proportional to the bifurcation buckling mode and had an amplitude equal to the thickness of the skin. The nonlinear analysis showed that the bifurcation approach in this case is not unduly unconservative and also that the presence of geometrical imperfections would not substantially reduce the critical load.

TABLE II

Data from Skylab Analysis

Number of Equations	20,910
Semi-bandwidth	252
Required Disc Storage	8.5×10^6
Total Run Time (cp)	2399 sec.
Time in Computer	3hr. 35 min.

BUCKLING ANALYSIS OF SPACE SHUTTLE CARGO DOORS

A bifurcation buckling analysis of the Space Shuttle cargo doors was performed by use of STAGS. A general view of the cargo doors is shown in Figure 13. There are five doors, each connected to the surrounding fuselage structure at discrete hinge and latch points, and to each other with shear pins. The door skin (sandwich) is stiffened by frames (see Fig. 14), and by longitudinal stiffeners at the hinge and latch edges. Intercostals are provided between frames at the edges of all doors. The doors are constructed from a composite material. Fuselage deformation results in loads on the doors at the hinges (applied displacement). In addition, there are loads due to aerodynamic pressure and gravity. The purpose of the STAGS analysis was to determine the bifurcation buckling load corresponding to a general instability mode. Therefore, the mesh spacing is relatively coarse, with 3 mesh spacings between frames, and 26 mesh spaces along the frames. This results in a model with a 106×27 node point mesh.

After initial runs on smaller models to determine mesh spacing requirements, etc., the complete 106×27 model was run. Two eigenvalues were found corresponding to the load factors; $\lambda = 7.007$ and $\lambda = 7.568$. The first eigenvalue, $\lambda = 7.007$ corresponds to a local buckle at the lower edge of the first (forward) door, in the flat part of the door, spilling over a little into the doubly curved region of the door. Due to the relatively coarse spacing, the buckling load for this local buckle is probably underestimated.

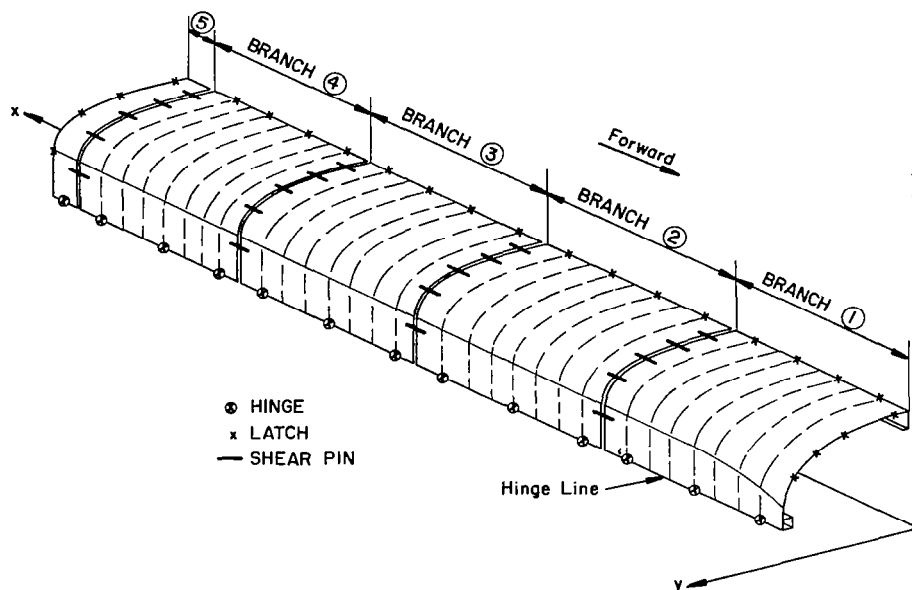


Fig. 13 Space Shuttle Cargo Doors

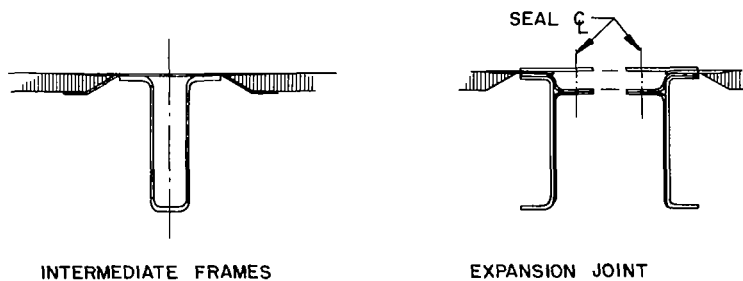


Fig. 14 Reinforcing Frames

The second eigenvalue, $\lambda = 7.568$, corresponds to a general instability mode in the last two doors (doors No. 4 and 5), and the singly curved region. The extent of the buckle is indicated in Figure 15. Several frames and intercostals take part in the buckle deformations.

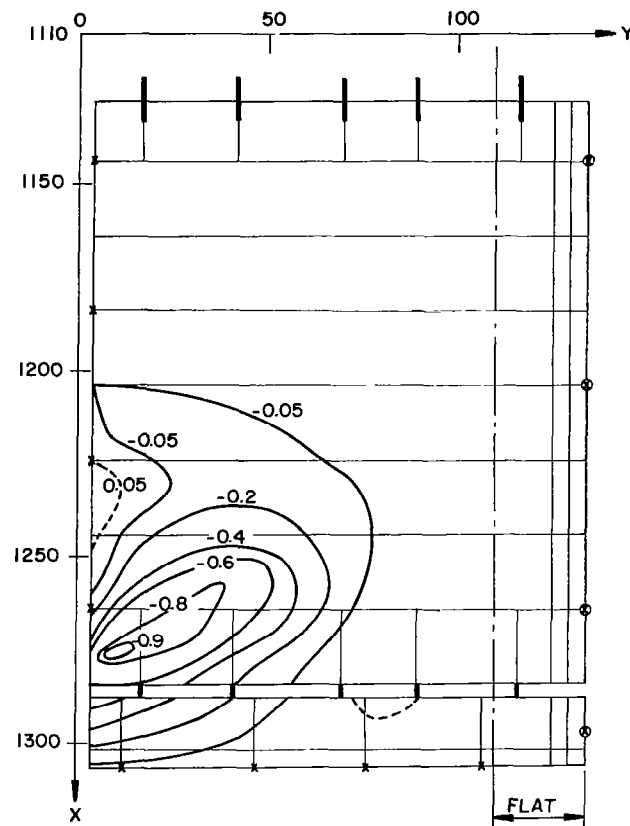


Fig. 15 Critical Buckling Mode for Cargo Doors

CONCLUDING REMARKS

The STAGS computer program is available to the general public as indicated in this report. It is primarily to be considered as a tool for analysis of shell or plate structures, although springs and beam elements also are included and three-dimensional elements are presently being introduced. Options are available for static stress, bifurcation buckling, vibration and transient response analysis. Geometric as well as material nonlinearities are included. The unique features of STAGS can be summarized as follows.

- Computational Efficiency - Much emphasis has been placed on efficient operation, which is particularly important in nonlinear analysis. STAGS gives rigorous solutions to the nonlinear equations and the runtime with STAGS appears to be very low in comparison to that of comparable codes.
- Generality - User written subroutines as an option to source data card inputs allow the user great generality in definition of structural geometry, loading, and geometric constraint conditions without the need for large numbers of input cards.
- Ease of Use - A STAGS input deck is usually compact and time for its preparation is short. Changes in grid spacing or computational strategy between consecutive runs on the same structure require very little work.

Many complicated structures consist of a number of simple components such as cylindrical panels, flat plates, etc. Such components (shell branches) can be defined separately with respect to geometry, material, loading, and mesh configuration. The use of "branched structures" is a major reason for the tractable input in STAGS. The numbering of the nodes becomes all automatic and internal constraints are defined on the branch level rather than on the element level. Another advantage of the use of branched structures is that different branches easily can be treated differently with respect to plasticity, geometric nonlinearity, output, etc.

The STAGS program is under development. Research is in progress at Lockheed MSC with the objective of making future versions of the program even more efficient and possibly easier to use. A mode of operation based on automatically computed global shape functions (Ritz functions) is now being prepared. Work is in progress on extension of the element library, including the introduction of three-dimensional elements and "contact elements".

REFERENCES

1. Wilkinson, J. H. and Reinsch, C., "Linear Algebra", Handbook for Automatic Computation, Volume II, Springer, 1971.
2. Almroth, B. O. and Felippa, C. A., "Structural Stability", Proceedings of International Symposium on Structural Mechanics Software, U. of Maryland, College Park, Maryland, June 1974, pp. 499-540.
3. Almroth, B. O. and Brogan, F. A., "Automated Choice of Procedures in Computerized Structural Analysis", presented at Second National Symposium on Computerized Structural Analysis and Design, Washington, D.C., 29-31 March 1976.
4. Gear, C. W., "Numerical Integration of Stiff Ordinary Differential Equations", Report 221, Dept. of Computer Sciences, U. of Illinois, Urbana, January 1967.
5. Jensen, P. S., "Stiffly Stable Methods for Undamped Second-Order Equations of Motion", SIAM J. Numer. Anal., Vol. 13, No. 4, pp. 549-563, September 1976.
6. Park, K. C., "An Improved Stiffly Stable Method for Direct Integration of Nonlinear Structural Dynamics", J. Appl. Mech., Vol. 42, 1975, pp. 464-470.
7. B. O. Almroth and Holmes, A. M. C., "Shells with Cutouts, Experiment and Analysis, International Journal Solids and Structures, Vol. 8, 1972, pp. 1057-1071.
8. Almroth, B. O., Brogan, F. A., and Marlowe, M. B., "Stability Analysis of Cylinders with Circular Cutouts", AIAA J., Vol. 11, 1973, pp. 1582-1584.
9. Stephens, W. B., Starnes, J. H., Jr., and Almroth, B. O., "Collapse of Longer Cylindrical Shells under Combined Bending and Pressure Loads", AIAA J., 13, 1975, pp. 20-25.
10. Stein, M. and Starnes, J. H., Jr., "Numerical Analysis of Stiffened Shear Webs in the Postbuckling Range", Numerical Solution of Nonlinear Structural Problems, AMD-Vol. 6, ASME 1973, pp. 211-223.

11. Almroth, B. O., Brogan, F. A., and Marlowe, M. B., "Collapse Analysis for Elliptic Cones", AIAA J., Vol. 9, 1971, pp. 32-37.
12. Marlowe, M. B., and Brogan, F. A., "Collapse of Elliptic Cylinders under Uniform External Pressure", AIAA J., Vol. 9, 1971, pp 2264-2266.
13. Skogh, J., Stern, P., and Brogan, F. A., "Instability Analysis of Skylab Structure", Computers and Structures, Vol. 3, 1973, pp. 1219-1240.
14. Skogh, J. and Stern, P., "Postbuckling Behavior of a Section Representative of the B-1 Aft Intermediate Fuselage", AFFDL TR-73-63, Wright-Patterson Air Force Base, Ohio, May 1973.
15. Skogh, J., Meller, E., and Brogan, F. A., "The Buckling and the Thereafter of the D-IT Stub Adapter", Lockheed Report LMSC/D358099, January 1974.
16. Brazier, L. G., "On the Flexure of Thin Cylindrical Shells and Other Thin Sections", Proc. of the Royal Soc., Series A., Vol. CXVI, 1926, pp. 104-114.
17. P. Seide and Weingarten, V. I., "On the Buckling of Circular Cylindrical Shells under Pure Bending", J. Appl. Mech., Vol. 28, 1961, pp. 112-116.
18. Ericsson, L. E., Almroth, B. O., Bailie, J. A., Brogan, F. A., and Stanley, G. M., "Hypersonic Aeroelastic Analysis", Lockheed Report LMSC/D056746, September 1975.
19. Heldenfels, R. R. and Vosteen, L. F., "Approximate Analysis of Effects of Large Deflections and Initial Twist on Torsional Stiffness of a Cantilever Plate Subjected to Thermal Stress", NACA TR 1361, 1958.

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16. Abstract The present document contains basic information about the computer code STAGS (Structural Analysis of General Shells). The purpose of the report is to describe to potential users the scope of the code and the solution procedures that are incorporated. Primarily, STAGS is intended for analysis of shell structures, although it has been extended to more complex shell configurations through the inclusion of springs and beam elements. The formulation is based on a variational approach in combination with local two-dimensional power series representations of the displacement components. STAGS includes options for analysis of linear or nonlinear static stress, stability, vibrations, and transient response. Material as well as geometric nonlinearities are included. A few examples of applications of the code are presented for further illustration of its scope.					
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